

Bago University
Department of Mathematics
First Semester Examination, March 2019

First Year (B.Sc)
(Mathematics Specialization)

Math 1101
Algebra and Analytic Geometry
Time Allowed: (3) hours

Answer All Questions.

1. (a) Verify, by truth table, that $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$ is a tautology.

(b) Negate each of the following statements:
(i) $\forall x, |x| = x$, (ii) $\exists x, x^2 = x$, (iii) $\exists x \forall y (p(x, y) \rightarrow q(x, y))$
(iv) $\exists y \exists x (p(x) \wedge \sim q(y))$.

2. (a) Solve for the exact roots of $4x^5 - 16x^4 + 17x^3 - 19x^2 + 13x - 3 = 0$.

(b) Find the value of $\sqrt{33}$ correct to four significant figures.

3. (a) Let n and r be non negative integers. Prove that $C(n, r) + C(n, r-1) = C(n+1, r)$.

(b) How many number of three different digits each less than 700 can be formed from the digits 1,2,3,4,5,6,7,8,9?

4. (a) By the method of mathematical induction, prove that $1 + 3 + 6 + \dots + \frac{n(n+1)}{2} = \frac{n(n+1)(n+2)}{6}$ is valid for all positive integral values of n .

(b) For what value of k will the line $3x + ky = 4$;
(i) have slope 2, (ii) have y -intercept 5, (iii) pass through $(-2, 4)$, (iv) be parallel to the line $2x - 5y = 1$, (v) be perpendicular to the line $4x + 3y = 2$?

5. (a) Prove that $(0, -2)$, $(-4, 8)$ and $(3, 1)$ lie on a circle with center $(-2, 3)$.

(b) Show that the curve $y^2 - 8x - 6y - 23 = 0$ is a parabola. Sketch its graph and show the focus and directrix.

6. (a) Sketch the hyperbola from the given curve $\frac{x^2}{16} - \frac{y^2}{4} = 1$, find the coordinates of the vertices and foci and find equation for the asymptote.

(b) Show that the curve $16x^2 + 9y^2 - 64x - 54y + 1 = 0$ is an ellipse. Sketch its graph and label the center, foci and the ends of the major and minor axes.
