Bago University
Department of Mathematics

Second Semester Examination, September 2019

Second Year(B.Sc) **Math 2107**

(Mathematics Specialization) **Linear Algebra I**

Time Allowed: (3) Hours

Answer All Questions.

1.(a) Show that the set of all (x, y, z) such that $x + y = 3z$ of elements in R^3 form a subspace.

(b) Let $c > 0$ be a rational number, and let γ be a real number such that $\gamma^2 = c$. Show that the set of all numbers which can be written in the form $a + b\gamma$, where a, b are rational numbers, is a field.

2.(a) Show that the vectors (1, 1) and (–1, 2) form a basis of $\mathbb{R}^2$.

(b) Find the coordinates of the vector X with respect to the vectors A, B, C.

$X = (0, 0, 1)$, $A = (1, 1, 1)$, $B = (-1, 1, 0)$, $C = (1, 0, -1)$.

3.(a) Let $V = K^3$ for some field K. Let W be the subspace generated by (1, 0, 0), and let U be the subspace generated by (1, 1, 0) and (0, 1, 1). Show that V is the direct sum of W and U.

(b) Let $F: V \to W$ be a linear map whose kernel is $\{0\}$. If $v_1, \ldots, v_n$ are linearly independent elements of V, prove that $F(v_1), \ldots, F(v_n)$ are linearly independent elements of W.

4.(a) Let $X = (0, 1, 0)$, and let A be an arbitrary 3×3 matrix. How would you describe XA? What if $X = (0, 0, 1)$? Generalize to similar statements concerning $n \times n$ matrices and their products with unit vectors.

(b) Let A be a square matrix. In general, if $A^n = O$ for some positive integer n, show that $I - A$ is invertible.

5.(a) Let F be the mapping defined by $F(x, y) = \left(\frac{x}{3}, \frac{y}{4}\right)$. What is the image under F of the ellipse $\frac{x^2}{9} + \frac{y^2}{16} = 1$?

(b) Determine which of the following mappings F are linear.

(i) $F: \mathbb{R}^4 \to \mathbb{R}^4$ defined by $F(X) = -X$.

(ii) $F: \mathbb{R}^2 \to \mathbb{R}$ defined by $F(x, y) = xy$.

6.(a) Let U, V, W be vector spaces over a field K. Let $F: U \to V$ be a linear mapping, and let G, H be two linear mappings of V into W. Prove that $(G + H) \circ F = G \circ F + H \circ F$.

(b) Show that the image under a linear map of a convex set is convex. Let $L: R^n \to R$ be a linear map and c be a number. Show that the set S consisting of all points A in R^n such that $L(A) > c$ is convex.

****