Bago University

Department of Mathematics

Second Semester Examination, September 2019

Second Year (B.Sc) **Math- 2109**

(Mathematics Specialization) **Vector Calculus and Dynamics**

Time Allowed: (3) hours

Answer All Questions.

1(a) A magnetic dipole of constant vector moment M is at the origin O and the magne-tostic potential U at P, where $\overline{OP}=\underline{r}$, is known to be given by $U=\frac{(\underline{M}.\underline{r})}{r^3}$.

If the magnetostatic field intensity at P is $\underline{H}=-\text{grand } U$, show that

$$\underline{H}=3r^{-5}(\underline{M}\bullet\underline{r})\underline{r}-r^{-3}\underline{M}.$$

(b) If ϕ_1,ϕ_2,ϕ_3 are uniform differentiable scalar functions and $\underline{F}$ a uniform differentiable vector function, prove that

(i) $\text{curl}(\phi\underline{F}) = \phi\,\text{curl}\,\underline{F} + (\text{grad}\phi)\times\underline{F}$

(ii) $\underline{\nabla}\bullet(\phi_1\underline{\nabla}\phi_2)=\phi_1\nabla^2\phi_2+\underline{\nabla}\phi_1.\underline{\nabla}\phi_2$.

2(a) At a given instant two cars are at distances 300 and 400 yd. from the point of intersection O of two straight roads crossing at a right angle and are approaching O at uniform speeds of 20 and 40 ft per sec. respectively. Find the shortest distance between the cars and the time taken to reach this position.

(b) A ship is steaming due east at 15m.p.h , and another ship is streaming due south at 20 m.p.h.; find the velocity of the second ship relative to the first.

3(a) The total mass of engine and train 200 tons, what is the horse power of the engine if it can just keep the train moving at a uniform speed of 60 m.p.h , on the level, the resistance due friction, etc., amounting to 10 lb, wt per ton?

(b) A man is cycling at 10 m.p.h. up a slope of 1 in 30. If the man and machine weight 180 lb. and frictional resistances are equivalent to 2 lb.wt., find the rate, in horse power, at which the man is working. Assuming that the man exerts a constant vertical pressure on each pedal in its downward path, find this pressure when the cranks are $6\frac{1}{2}$ in. long and the gear is 72in.

4(a) Find the horse-power required to pump 1000 gals. of water per min. from a depth of 50 ft. and deliver it through a pipe of 6 sq.in, cross section. (Assume that 1 cu.ft of water is 6.5 gal. and that 1 gal. weighs 10 lb. and neglect the effects of friction.)

(b) A car is driven at a uniform speed of 30 m.p.h up an incline of 1 in 8. If the local weight of the car is 1850 lb., and resistances are neglected, calculate the horse power at which the car is working.

P.T.O.

5.(a) The resistance which a train experiences, when moving at V m.p.h., is equal to $(6+\frac{V^2}{100})$ lb.wt per ton weight of the train. If a train of weight 150 tons is drawn up a slope of 1 in 140 by an engine of 372 h.p. Show that the maximum speed attainable is 30 m.p.h. Also find the maximum speed down the slope with stream shut off.

(b) A particle is moving with S.H.M , in a straight line and takes 3 sec. to perform a complete oscillation. Its furthest distance from the centre of force is 4 ft. Find its maximum acceleration and maximum velocity. If , when at its furthest point , it receives a blow which drives it in with an initial velocity of u ft per sec. find its new amplitude. What value of u will make the amplitude 5 ft. instead of 4ft ?

6.(a) At the ends of three successive seconds the distances of a point moving with S.H.M. from its means position, measured in the same direction, are 1,5 and 5. Show that the period of the complete oscillation is $\frac{2\pi}{\cos^{-1}\left(\frac{3}{5}\right)}$ sec. sec.

(b) A light string passing through a smooth ring at O on a smooth horizontal table has particles each of mass m attached to its ends A and B. Initially the particles lie on the table with the portions of the string OA and OB stringht, and OA =OB. An impulse P is applied to the particle A in a direction making 60° with OA. Prove that when B reaches O its velocity is $\frac{P}{m}\cdot\frac{\sqrt{22}}{8}$.

****