Bago University
Department of Mathematics
Second Semester Examination, September 2019

Second Year (B. Sc) **Math 2111**
(Mathematics Specialization) **Theory of Sets II**
Answer All Questions. **Time Allowed: (3) Hours**

1.(a) Let S be a subset of a well-ordered set A with the following properties:

(i) $a_0 \in S$

(ii) $s(a) \subset S$ implies $a \in S$.

Prove that $S = A$.

(b) Prove that a well-ordered set cannot be similar to one of its initial segments.

2.(a) Let A be a well-ordered set, let B be a subset of A, and let the function $f : A \to B$ be a similarity mapping of A into B. Prove that $a \precsim f(a)$ for every $a \in A$.

(b) Define the ordinal number of a well-ordered set. Let $\lambda = \text{ord}\ (A)$ and let $\mu < \lambda$.

Prove that there is a unique initial segment $s(a)$ of A such that $\mu = \text{ord}\ (s(a))$.

3.(a) Let A be a well-ordered set and let S be a subset of A with the properties that $a \precsim b$ and $b \in S$ implies $a \in S$. Prove that $S = A$ or S is an initial segment of A.

(b) Suppose that A is a well-ordered set and B is similar to A. Prove that B is a well-ordered set.

4.(a) Let A and B be well-ordered sets and let

$$S = \left\{ x \,\middle|\, x \in A,\ s(x) \simeq s(y) \text{ where } y \in B \right\}$$

and $T = \left\{ y \,\middle|\, y \in B,\ s(y) \simeq s(x) \text{ where } x \in A \right\}$. Prove that $S \simeq T$.

(b) Prove by giving a counter-example, that the operation of addition for ordinal numbers is not commutative.

5.(a) For any ordinal number ω, prove that $\omega + \omega = \omega 2$ with and without using the left distributive law.

(b) Let λ be any ordinal number. Prove that $\lambda + 1$ is the immediate successor of λ.

6.(a) Prove that the Axiom of Choice is equivalent to Zermelo's Postulate.

(b) Let B be a partially ordered set. Prove that there exists a totally ordered subset A of B such that A is not a proper subset of any other totally ordered subset of B.