Bago University
Department of Mathematics
First Semester Examination, March 2019

Third Year (B.Sc)
(Mathematics Specialization)

Math 3101
Analysis I
Time Allowed: (3) hours

Answer All Questions.

1.(a) Define countable set and finite set. Let Z denote the set of all integers. Prove that Z is infinite and countable.

(b) Let $\{E_n\}, n = 1,2,3, \ldots$ be a sequence of countable sets. Prove that countable union of countable sets is countable.

2.(a) Define a limit point and an interior point. Let $\{E_\alpha\}$ be a finite or infinite collection of sets E_α. Show that $\left(\bigcup_\alpha E_\alpha\right)^c = \bigcap_\alpha (E_\alpha^c)$.

(b) Show that a set E is open if and only if E^c is closed.

3.(a) If $\{K_\alpha\}$ is a collection of compact subsets of a metric space X such that the intersection of every finite subcollection of $\{K_\alpha\}$ is nonempty, prove that $\bigcap K_\alpha$ is nonempty.

(b) Define a compact set. Prove that closed subsets of compact sets are compact.

4.(a) Let $\{p_n\}$ be a sequence in a metric space X with metric d. If $p, p' \in X$ and if $\{p_n\}$ converges to p and to p', prove that $p' = p$.

(b) Suppose $\{s_n\}$ is a complex sequence and $\lim_{n\to\infty} s_n = s$. Verify that $\lim_{n\to\infty} \frac{1}{s_n} = \frac{1}{s}$ if $s \neq 0$, $s_n \neq 0$ $(n = 1, 2, 3, \ldots)$.

5.(a) Define a diameter. Let $\overline{E}$ be the closure of a set E in a metric space X. Prove that diam $\overline{E}$ = diam E.

(b) Let $\{s_n\}$ be a sequence of real numbers. Suppose $\{s_n\}$ is monotonic. Prove that $\{s_n\}$
convergences if and only if it is bounded.

6.(a) State and prove the Comparison Test.

(b) Find the radius of converges of each of the following power series:

(i) $\sum \frac{2^n}{n^2} z^n$, (ii) $\sum \frac{n^3}{3^n} z^n$.

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