Bago University

Department of Mathematics

First Semester Examination, March 2019

Third Year (B.Sc)

Mathematics Specialization

Math 3102

Linear Algebra II

Time Allowed: (3) hours

Answer All Questions.

1.(a) Let V, W be vector spaces over K, and let $F : V \to W$ be a linear map. Let $\mathcal{B}$ be a basis of V and $\mathcal{B}'$ a basis of W. If $v \in V$, prove that

$X_{\mathcal{B}'}(F(v)) = M^{\mathcal{B}}_{\mathcal{B}'}(F)X_{\mathcal{B}}(v)$.

(b) Find the matrix associated with the following linear maps:

(i) $F : R^4 \to R^2$ given by $F({}^t(x_1, x_2, x_3, x_4)) = {}^t(x_1, x_2)$ (the projection);

(ii) $F : R^n \to R^n$ given by $F(X) = 7X$.

2.(a) Find the matrix $R(\theta)$ associated with the rotation for the values of θ.

(i) $\pi/2$ (ii) $-\pi/3$.

(b) Let $X = {}^t(1,2)$ be a point of the plane. Let F be the rotation through an angle of $\pi/4$. What are the coordinates of $F(X)$ relative to the usual basis $\{E^1, E^2\}$?

3.(a) Let F be the rotation through an angle θ. Let (x,y) be a point of the plane in the standard coordinate system. Let (x',y') be the coordinates of this point in the rotated system. Express x', y' in terms of x , y and θ.

(b) Let $D = d/dt$ be the derivative and $\mathcal{B}$, a set of linearly independent functions , generate a vector space V. D is a linear map from V into itself. Find the matrix associated with D relative to the bases $\mathcal{B}, \mathcal{B}$.

(i) $\{e^t, e^{2t}\}$, (ii) $\{1, t\}$.

4.(a) Let c be a number and let A be an $n \times n$ matrix. Show that $D(cA) = c^n D(A)$.

(b) Let $f(t)$, $g(t)$ be two functions having derivatives of all orders. Let $\varphi(t)$ be the function obtained by taking the determinant $\varphi(t) = \begin{vmatrix} f(t) & g(t) \\ f'(t) & g'(t) \end{vmatrix}$.

Show that $\varphi'(t) = \begin{vmatrix} f(t) & g(t) \\ f''(t) & g''(t) \end{vmatrix}$.

P.T.O

ne following system of linear equations by Cramer's Rule.

$$\begin{aligned} 4x+y+z+\ \ w &= 1 \\ x-y+2z-3w &= 0 \\ 2x+y+3z+5w &= 0 \\ x+y-z-\ \ w &= 2. \end{aligned}$$

ne the sign of the following permutation.

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}.$$

inverse of the following matrix.

$$\begin{pmatrix} 3 & -1 & 5 \\ -1 & 2 & 1 \\ -2 & 4 & 3 \end{pmatrix}.$$

e the rank of the following matrix.

$$\begin{pmatrix} 3 & 5 & 1 & 4 \\ 2 & -1 & 1 & 1 \\ 7 & 1 & 2 & 5 \end{pmatrix}.$$

****