Bago University

Department of Mathematics

Second Semester Examination, September 2019

Third Year (B.Sc) **Math-3107**

(Mathematics Specialization) **Analysis II**

Answer All Questions. **Time Allowed: (3) hours**

1.(a) Let X and Y be metric spaces. Suppose $E \subseteq X$, f maps E into Y and p is a limit point of E. Prove that $\lim_{x\to p} f(x) = q$ if and only if $\lim_{n\to\infty} f(p_n) = q$ for every sequence $\{ p_n \}$ in E such that $p_n \neq p$ $\lim_{n\to\infty} p_n = p$.

(b) Suppose X, Y, Z are metric spaces, $E \subset X$, f maps E into Y, g maps the range of f, $f(E)$ into Z and h is a mapping of E into Z defined by $h(x) = g(f(x))$ $(x \in E)$. If f is continuous at a point $p \in E$ and g is continuous at the point $f(p)$, prove that h is continuous at p.

2.(a) Let f and g be complex continuous functions on a metric space X. Prove that

$f+g,\ fg,\ \frac{f}{g}$ are continuous on X.

(b) Define a continuous function.

Prove that a mapping f of a metric space X into a metric space Y is continuous if and only if $f^{-1}(E)$ is closed in X for every closed set E in Y.

3.(a) Let X be a metric space. Suppose $E \subseteq X$, p is a limit point of E, f and g are complex functions on E and $\lim_{x\to p} f(x) = A$, $\lim_{x\to p} g(x) = B$. Prove that

$$\lim_{x\to p}(f+g)(x) = A + B.$$

(b) Prove that the function $f:\mathbb{R}\to\mathbb{R}$ is defined by $f(x) = \begin{cases} x, & x \text{ is rational} \\ 0, & x \text{ is irrational} \end{cases}$

is continuous at $x = 0$.

P.T.O.

4.(a) Suppose f is continuous one-one mapping of a compact metric space X onto a metric space Y. Prove that the inverse mapping f^{-1} defined on Y by $f^{-1}(f(x)) = x$ $(x \in X)$ is a continuous mapping of Y onto X.

(b) Let f be defined on $[a,b]$. If f has a local maximum at a point $x \in (a,b)$ and if $f'(x)$ exists, prove that $f'(x) = 0$.

5.(a) If f and g are continuous real functions on $[a,b]$ which are differentiable in (a,b), then prove that there is a point $x \in (a,b)$ at which,

$$[f(b) - f(a)]\, g'(x) = [g(b) - g(a)]\, f'(x).$$

(b) Suppose f is a real differentiable function on $[a,b]$ and suppose $f'(a) < \lambda < f'(b)$. Prove that there is a point $x \in (a,b)$ such that $f'(x) = \lambda$.

6.(a) Give an example that L'Hospital rule fails for complex function.

(b) Let f be defined for real x and suppose that $|f(x) - f(y)| \le (x-y)^2$ for all real x and y. Show that f is constant.

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