Bago University
Department of Mathematics
First Semester Examination, March 2019

Fourth Year (B.Sc)
(Mathematics Specialization)

Math 4104
Partial Differential Equations
Time Allowed: (3) hours

Answer All Questions.

1.(a) Show that the given function $u = e^y f(x-y)$ satisfy the equation $u = u_x + u_y$.

(b) Show that if $u = u(x, y)$ and $v = v(x, y)$ satisfy Cauchy-Riemann equations $u_x = v_y,\ u_y = -v_x$ then each is a solution of Laplace's equation.

2.(a) Solve the simultaneous differential equation : $\dfrac{dx}{mu-ny} = \dfrac{dy}{nx-lu} = \dfrac{du}{ly-mx}$.

(b) Solve the partial differential equation $y^2 u_x - xyu_y = x(u-2y)$.

3. Consider the Tricomi equation : $u_{xx} + xu_{yy} = 0,\ \ x < 0$. Find a mapping $q = q(x, y),\ r = r(x, y)$, that transform the equation into its canonical form, and present the equation in this coordinate system.

4.(a) Expand $f(x) = x^2\ (-\pi \le x \le \pi)$ in a Fourier series.

(b) Expand $f(x) = Ax^2 + Bx + C\ (0 < x < 2\pi)$, where A, B and C are constants, in Fourier series.

5.(a) Expand $f(x) = \cos ax (-\pi \le x \le \pi)$, where a is not an integers, in Fourier series.

(b) Expand in Fourier series :

$$f(x) = \begin{cases} 0, & -\pi < x < 0 \\ x, & 0 \le x < \pi. \end{cases}$$

6. Solve the equation $u_t = 17u_{xx},\ 0 < x < \pi,\ t > 0$, with the boundary conditions $u(0,t) = u(\pi,t) = 0,\ t \ge 0$, and the initial conditions

$$u(x,0) = \begin{cases} 0, & 0 \le x \le \dfrac{\pi}{2} \\ 2, & \dfrac{\pi}{2} < x \le \pi. \end{cases}$$

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