Bago University
Department of Mathematics
First Semester Examination, March 2019

Fourth Year (B.Sc)
(Mathematics Specialization)

Math 4105
Stochastic Process I
Time Allowed: (3) hours

Answer All Questions.

1.(a) The joint probability mass function of X and Y, $p(x, y)$, is given by $p(1,1)=\frac{1}{9}, p(2,1)=\frac{1}{3}, p(3,1)=\frac{1}{9}$. Compute $E[X|Y=1]$.

(b) If X and Y are independent Poisson random variables with respective means λ_1 and λ_2, calculate the conditional probability mass function of X given that $X+Y=n$.

2.(a) The joint density of X and Y is given by

$$f(x,y)=\begin{cases}\frac{1}{2}ye^{-xy}, & 0<x<\infty, 0<y<2\\ 0, & \text{otherwise.}\end{cases}$$

What is $E[e^{\frac{x}{2}}|Y=1]$?

(b) If X and Y are discrete random variables, prove that $E[X]=\sum_{y} E[X|Y=y]P\{Y=y\}$.

3.(a) A miner is trapped in a mine containing three doors. The first door leads to a tunnel which takes him to safety after two hours of travel. The second door leads to a tunnel which returns him to the mine after three hours of travel. The third door leads to a tunnel which returns him to his mine after five hours . Assuming that the miner is at all times equally likely to choose any one of the doors, what is the expected length of time until the miner reaches safety ?

(b) Show in the discrete case that if X and Y are independent, show that $E[X|Y=y]=E[X]$ for all y.

4.(a) Suppose a Markov chain with three states has the probability transition matrix

$$P=\begin{pmatrix}0 & 1 & 0\\ 0.5 & 0 & 0.5\\ 1 & 0 & 0\end{pmatrix}.$$

Determine whether or not this Markov chain is irreducible.

(b) A raining process is considered as a two state Markov chain. If it rains, it is considered to be an state "0" and if it does not rain, the chain is in state "1". The transition probability of the Markov chain is defined as

$P=\begin{pmatrix}0.6 & 0.4\\ 0.2 & 0.8\end{pmatrix}$. Find the probability that it will rain for 3days from today assuming that it is raining. Also find the unconditional probability that it will rain after 3 days of the state "0" and "1" as 0.4 and 0.6 respectively.

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5. (a) Consider a communication system that transmits the digit '0' and '1' through several stage. At each stage the probability that the same digit will be received by the next stage, as transmitted is 0.75. What is the probability that a '0' that is entered the first stage is received as '0' by the fifth stage?

(b) A Markov chain $\{X_n: n \geq 0\}$ with states 0,1,2 has the probability transition matrix

$$P = \begin{pmatrix} 1/2 & 1/3 & 1/6 \\ 0 & 1/3 & 2/3 \\ 1/2 & 0 & 1/2 \end{pmatrix}.$$ If $P(X_0 = 0) = P(X_0 = 1) = \frac{1}{4}$. Find $E[X_3]$.

6. (a) Consider a Markov chain having state 0,1,2,3,4 transition probability matrix and

$$P = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{2} \end{pmatrix}.$$

What are the communication classes? Which classes are recurrent and which are transient?

(b) Suppose that the chance of rain tomorrow depends on previous weather conditions only through whether or not it is raining today and not on past weather conditions. Assume that if it rains today, then it will rain tomorrow with probability $\alpha = 0.7$ and if does not rain today, then it will rain tomorrow with probability $\beta = 0.4$. If we say that the state is 0 when it rains and 1 when it does not rain. Find the limiting probability of rain. Find the limiting probability of not rain.

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