Bago University

Department of Mathematics

Second Semester Examination, September 2019

Fourth Year (B.Sc) **Math-4108**

(Mathematics Specialization) **General Topology-I**

Answer All Questions. **Time Allowed: (3) Hours**

1. (a) Let $p \in G$, an open subset of $\mathbb{R}^2$. Show that there exists an open disc D with center p such that $p \in D \subset G$.

   (b) Prove that a subset A of $\mathbb{R}^2$ is closed if and only if A contains each of its accumulation points.

2. (a) Let $\langle a_n \rangle$ be a Cauchy sequence. If a subsequence $\langle a_{i_n} \rangle$ of $\langle a_n \rangle$ converges to a point b, show that the Cauchy sequence itself converges to b.

   (b) Show that a function $f : \mathbb{R}^2 \to \mathbb{R}^2$ is continuous if and only if the inverse image of every open set is open.

3. (a) Let $\{\tau_i : i \in I\}$ be any collection of topologies on a set X. Prove that the intersection $\bigcap_i \tau_i$ is also a topology on X and also show that the union of topologies need not be a topology.

   (b) The class $\tau = \{X, \emptyset, \{a\}, \{c, d\}, \{a, c, d\}, \{b, c, d, e\}$ defines a topology on X. Consider the subset $A = \{a, b, c\}$ of X. Find the derived set of A with calculation.

4. (a) Let A and B be subsets of a topological space (X, τ). Show that
$$(A \cup B)' = A' \cup B'.$$

   (b) Let $\tau = \{X, \phi, \{a\}, \{a,b\}, \{a,c,d\}, \{a,b,c,d\}, \{a,b,e\}\}$ be a topology on $X = \{a,b,c,d,e\}$:

   (i) Find the interior points of the subset $A = \{a,b,c\}$ of X.

   (ii) Find the exterior points of A.

   (iii) Find the boundary points of A.

5. (a) Let A be any subset of a topological space X. Show that the closure of A is the union of the interior and boundary of A.

   (b) Let $\mathcal{B}$ and $\mathcal{B}^*$ be base for topologies τ and τ^* on a set X respectively. Suppose that each $B \in \mathcal{B}$ is the union of members of $\mathcal{B}^*$. Show that $\tau \subset \tau^*$.

6. (a) Prove that a point p in a topological space X is an accumulation point of $A \subset X$ if and only if each member of some local base $\mathscr{B}_p$ at p contains a point of A different from p.

   (b) Let $X = \{a,b,c,d,e\}$ and let $\mathcal{A} = \{\{a,b,c\}, \{c,d\}, \{d,e\}\}$. Find the topology on X generated by $\mathcal{A}$.

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