Bago University
Department of Mathematics
Second Semester Examination, September 2019

Fourth Year (B. Sc)
(Mathematics Specialization)

Math 4109
Abstract Algebra I
Time Allowed: (3) Hours

Answer All Questions.

1.(a) Define a group. Let $\mathbb{R}$ be the set of real numbers. Let K be the set of all mappings $T_{1,b} : \mathbb{R} \to \mathbb{R}$ defined by $T_{1,b}(r) = r + b$ for each $r \in \mathbb{R}$ and for some $b \in \mathbb{R}$. Let the operation $*$ be the product of two mappings in K which gives,

$$\begin{aligned}(T_{1,a} * T_{1,b})(r) &= T_{1,a}(T_{1,b}(r)) \\ &= T_{1,b}(r) + a \\ &= (r + b) + a \\ &= r + (a + b) = T_{1,a+b}(r).\end{aligned}$$

That is, $T_{1,a} * T_{1,b} = T_{1,a+b}$. Prove that K is a group under $*$.

(b) Let G be a group in which $(ab)^3 = a^3 b^3$ and $(ab)^5 = a^5 b^5$ for all $a, b \in G$. Show that G is an abelian group.

2.(a) Let H be a nonempty subset of a group G such that $ab^{-1} \in H$ whenever given $a, b \in H$. Prove that H is a subgroup of G.

(b) Define a subgroup. Let G be any group and let $C(a)$ be the centralizer of a in G defined by

$$C(a) = \{ g \in G : ga = ag \} \text{ for } a \in G.$$

Prove that $C(a)$ is a subgroup of G.

3.(a) Let G be a group and φ a homomorphism of G onto G' and K be the kernel of φ. Prove that K is a normal subgroup of G.

(b) Let M be a subgroup of a group G such that $x^{-1}Mx \subset M$ for all $x \in G$. Prove that $x^{-1}Mx = M$.

4.(a) Define a cyclic group. Suppose that a group G is cyclic group, show that every subgroup of G is cyclic group.

(b) Define an equivalence relation. Let G be a group. Suppose that there is a relation $\sim$ on G defined by $a \sim b$ if there exists an $x \in G$ such that $b = x^{-1}ax$. Prove that the relation $\sim$ is an equivalence relation on G.

5.(a) Let G be a group such that $a^5 = e$ and $aba^{-1} = b^2$ for $a, b \in G$. Find $o(b)$ if $b \neq e$.

(b) If G is abelian and $\varphi : G \to G'$ is a homomorphism G onto G', prove that G' is abelian.

P.T.O.

6.(a) Let N be a normal subgroup of an abelian group G. Prove that the quotient group of G by N is abelian.

(b) Suppose that an abelian group G has an element of order m and one of order n, where m and n are relatively prime. Prove that G has an element of order mn.

****