**First Year (B.Sc)**                                                          **Math 1102**
**(Mathematics Specialization)**                     **Trigonometry and Differential Calculus**
                                                                  **Time Allowed: (3) hours**

**Answer All Questions.**

1. ( a ) Find the slope of the curve $y = x^3 - 12x$ at the given point $P(1, -11)$ and an equation of tangent line to the curve at this point.

   ( b ) Find the limits; (i) $\lim_{h \to 0} \frac{\sqrt{5h+4}-2}{h}$ ,        (ii) $\lim_{x \to 4} \frac{4-x}{5-\sqrt{x^2+9}}$.

2. ( a ) Using the Sandwich Theorem, if $2 - x^2 \le g(x) \le 2\cos x$ for all $x$, find $\lim_{x \to 0} g(x)$.

   ( b ) Define $f(1)$ in a way that extends $f(s) = \frac{s^3-1}{s^2-1}$ to be continuous at $s = 1$.

3. ( a ) Show that    (i) $\lim_{h \to 0} \frac{\cos h - 1}{h} = 0$          (ii) $\lim_{x \to 0} \frac{\sin 2x}{5x} = \frac{2}{5}$.

   ( b ) Let $f(x) = 2x^2, x \ge 0$ find the value of $f^{-1}(x)$. Evaluate $\frac{df}{dx}$ at $x = 5$ and $\frac{df^{-1}}{dx}$ at $x = f(5)$, show that $\frac{df^{-1}}{dx} = \frac{1}{\frac{df}{dx}}$ at this point.

4. ( a ) Use by logarithmic differentiation, find $\frac{dy}{dt}$ if $y = \frac{(x^2+1)(x+3)^{\frac{1}{2}}}{x-1}, x > 1$.

   ( b ) Find $\frac{dy}{dx}$ or $\frac{dy}{dt}$ if (i) $y = t\sqrt{\ln t}$          (ii) $y = 3^{\sin x}$          (iii) $y = \tan^{-1}\sqrt{x+1}$.

5. ( a ) Use by L'Hospital's rule, find    ( i ) $\lim_{x \to \infty}(\ln x)^{\frac{1}{x}}$,    ( ii ) $\lim_{\theta \to \frac{\pi}{2}} \frac{2\theta - \pi}{\cos(2\pi - \theta)}$.

   ( b ) Use the definitions of $\cosh x$ and $\sinh x$ to show that
        ( i ) $\sinh 2x = 2\sinh x \cosh x$,        ( ii ) $\cosh^2 x - \sinh^2 x = 1$.

6. ( a ) Find the Taylor series generated by $f(x) = x^4 + x^2 + 1$ at $a = -2$.

   ( b ) Find the Maclaurin series for the function $f(x) = e^{-x}$.

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