**Bago University**

**Department of Mathematics**

**Second Semester Examination, September 2019**

**First Year (B.Sc)** **Math 1103**

**(Mathematics Specialization)** **Algebra and Analytical Solid Geometry**

**Answer All Questions.** **Time Allowed: (3) Hours**

**1.(a)** Find all solutions to the given linear system

$$\begin{aligned} 2x - y + z &= 3 \\ x - 3y + z &= 4 \\ -5x \quad - 2z &= -5 \end{aligned}$$ by Gauss Jordan reduction.

**(b)** Determine the following linear system has a nontrivial solution by using singular or non-singular test.

$$\begin{aligned} 2x + y - z &= 0 \\ x - 2y - 3z &= 0 \\ -3x - y + 2z &= 0. \end{aligned}$$

**2.(a)** Verify the following (do not expand the determinant) using the properties of determinants.

$$\begin{vmatrix} a-b & 1 & a \\ b-c & 1 & b \\ c-a & 1 & c \end{vmatrix} = \begin{vmatrix} a & 1 & b \\ b & 1 & c \\ c & 1 & a \end{vmatrix}.$$

**(b)** Solve the linear system by Cramer's rule.

$$\begin{aligned} 2x + 4y + 6z &= 2 \\ x \qquad + 2z &= 0 \\ 2x + 3y - z &= -5. \end{aligned}$$

**3.(a)** Show that (i) $i(1-\sqrt{3}i)(\sqrt{3}+i) = 2(1+\sqrt{3}i)$.

(ii) $5i/(2+i) = (1+2i)$.

**(b)** Use de Moivre's formula to derive the following trigonometric identities:

(i) $\cos 3\theta = \cos^3\theta - 3\cos\theta\sin^2\theta$; (ii) $\sin 3\theta = 3\cos^2\theta\sin\theta - \sin^3\theta$.

**P.T.O.**

**4.(a)** Graph the sets of points whose polar coordinates satisfy the following conditions.

(i) $1 \leq r \leq 2$ and $0 \leq \theta \leq \pi/2$

(ii) $-3 \leq r \leq 2$ and $\theta = \pi/4$

(iii) $2\pi/3 \leq \theta \leq 5\pi/6$ ( no restriction on r).

**(b)** Find the area of the region in the plane enclosed by the cardioid $r = 2(1+\cos\theta)$.

**5.(a)** Find the ratio in which the line joining the points (2,4,5), (3,5,−4) is divided by the YZ plane.

**(b)** Find the equation of the plane through the points (2,2,1) and (9,3,6) and perpendicular to the plane $2x + 6y + 6z = 9$.

**6.(a)** Show that the line $\frac{x-3}{3} = \frac{2-y}{4} = \frac{z+1}{1}$ intersects the line $x + 2y + 3z = 0$ and $2x + 4y + 3z + 3 = 0$. Find their point of intersection.

**(b)** Find the image of the point $P(1, 3, 4)$ in the plane $2x - y + z + 3 = 0$.

**********