Bago University
Department of Mathematics
First Semester Examination, March 2019

Second Year (B.Sc) **Math-2101**

Mathematics Specialization **Complex Variables I**

Time Allowed:(3) Hours

Answer All Questions.

1.(a) Prove that $\lim_{z \to z_0} \operatorname{Re} z = \operatorname{Re} z_0$.

(b) Show that the limit of the function $f(z) = \left(\frac{z}{\bar{z}}\right)^2$ as tends to 0 does not exist.

2.(a) Verify the function $f(z) = \cosh x \cos y + i \sinh x \sin y$ is entire.

(b) Show that the function $u(x, y) = 2x - x^3 + 3xy^2$ is harmonic in some domain and find a harmonic conjugate $v(x, y)$.

3.(a) Write $|\exp(2z+i)|$ and $|\exp(iz^2)|$ in terms of x and y. Then show that

$$\left|\exp(2z+i)+\exp(iz^2)\right| \le e^{2x} + e^{-2xy}.$$

(b) Find the principal value of $(1 - i)^{2i}$ and show that

$$\log(-1 + \sqrt{3}\, i) = \ln 2 + 2\left(n + \frac{1}{3}\right)\pi i, \ (n = 0, \pm 1, \pm 2, \ldots).$$

4.(a) Show that $\log(i^2) \neq 2 \log i$ when $\log z = \ln r + i\theta$, $\left(r > 0, \frac{3\pi}{4} < \theta < \frac{11\pi}{4}\right)$.

(b) Let C_1 denote the right half of the circle $z = 2e^{i\theta}$ $\left(-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}\right)$ and let C_2 denote the left half of the same circle $z = 2e^{i\theta}$ $\left(\frac{\pi}{2} \le \theta \le \frac{3\pi}{2}\right)$. Evaluate $\int_C \frac{dz}{z}$, where $C = C_1 + C_2$.

5.(a) Let C be the circle $|z| = 3$, described in the positive sense. Show that if $g(z) = \int_C \frac{2s^2 - s - 2}{s - z}\, ds$ $(|z| \neq 3)$,then $g(2) = 8\pi i$ .What is the values of $g(z)$ when $|z| > 3$?

(b) If C denotes any closed contour lying in the open disk $|z| < 2$, prove that

$$\int_C \frac{z e^z}{(z^2+9)^5}\, dz = 0.$$

6.(a) Evaluate the integral $\int_C \frac{5z-2}{z(z-1)}\, dz$,where C is the circle $|z| = 2$ described counterclockwise.

(b) Suppose that $f(z) = \frac{(\log z)^3}{z^2+1}$, where $\log z = \ell n\, r + i\theta$ $(r > 0, 0 < \theta < 2\theta)$. Find the residue of f and pole of order. ■