**Bago University**

**Department of Mathematics**

**First Semester Examination, March 2019**

**Second Year (B.Sc)** **Math 2104**

**Mathematics Specialization** **Discrete Mathematics I**

**Time Allowed: (3) Hours**

**Answer All Questions.**

1.( a ) The letter ABCDE are to be used to form strings of length 3.

(i) How many strings can be formed if we allow repititions ?

(ii) How many strings do not contain the letter A, allowing repititions ?

(iii) How many strings contain the letter A, allowing repititions ?

( b ) How many(unordered) five-card poker hands, selected from an ordinary 52-card deck, are there ?

(i) How many poker hands contain cards all of the same suit ?

(ii) How many poker hands contain three cards of one denomination and two cards of a second denomination ?

2.( a ) In a club consisting of six distinct men and seven distinct women,

(i) how many ways can we select a committee of five persons?

(ii) how many ways can we select a committee of four persons that has at least one woman ?

( b ) Find the 6-combination that will be generated by Algorithm after 145678 if $n = 8$.

3.( a ) Piles of identical red, blue, and green balls where each pile contains at least 10 balls.

(i) In how many ways can 10 balls be selected if at least one red ball must be selected ?

(ii) In how many ways can 10 balls be selected if at least one red ball, at least two blue balls and at least three green balls must be selected ?

( b ) Find the number of integer solutions of $x_1 + x_2 + x_3 = 15$ subject to the given conditions ?

(i) $x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$

(ii) $x_1 = 1, x_2 \geq 0, x_3 \geq 0.$

**P.T.O**

4.( a ) Suppose that $n$ is even. Prove that

$$\sum_{k=0}^{\frac{n}{2}} C(n,2k) = 2^{n-1} = \sum_{k=1}^{\frac{n}{2}} C(n,2k-1) \ .$$

(b) An inventory consists of a list of 115 items, each marked "available" or "unavailable". There are 60 available items. Show that there are at least two available items in the list exactly four items apart.

5.(a) Assume that a person invests \$2000 at 14 percent compounded quarterly. Let $A_n$ represent the amount at the end of $n$ years.

(i) Find the recurrence relation for the sequence $A_0, A_1, \ldots$ .

(ii) Find the initial condition for the sequence $A_0, A_1, \ldots$ .

(iii) Find $A_1, A_2, A_3$.

(iv) Find the explicit formula for $A_n$.

(v) How long will it take for a person to double the initial investment ?

( b ) Solve the recurrence relation $p_n = a - \frac{b}{k} p_{n-1}$ by iteration.

6.( a ) Solve the recurrence relation $c_n = 2c_{n-1} + 1$ , subject to the initial condition $c_1 = 1$.

( b ) Find the general solution of the recurrence relation $a_n = 7a_{n-1} - 10a_{n-2} + 16n$.

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