Bago University

Department of Mathematics

First Semester Examination, March 2019

Second Year (B.Sc)
(Mathematics Specialization)

Math 2105
Theory of Sets I
Time Allowed: (3) hours

Answer All Questions.

1.(a) Define the equivalent sets.

Suppose that $N = \{1, 2, 3, \ldots\}$ and $E = \{2, 4, 6, \ldots\}$. Prove that N is equivalent to E.

(b) Prove that $[0,1] \sim (0,1)$.

2.(a) Show that $\left\{1, \frac{1}{2}, \frac{1}{3}, \ldots, \frac{1}{n}, \ldots\right\}$, set of infinite sequence, is denumerable.

(b) Let a and b be any two real numbers with $a < b$. Prove that the closed interval $[a,b]$ has the power of continuum and has the cardinality c.

3.(a) Prove that every subset of a denumerable set is either finite or denumerable.

(b) For any cardinal numbers α, β, γ, prove that (i) $\alpha + \beta = \beta + \alpha$,

(ii) $(\alpha\beta)\gamma = \alpha(\beta\gamma)$ and

(iii) $\alpha(\beta + \gamma) = \alpha\beta + \alpha\gamma$.

4.(a) Define a partial order in a set.

Let $\mathcal{A}$ be the family of sets. Prove that the relation in $\mathcal{A}$ defined by "x is a subset of y" is a partial order in $\mathcal{A}$.

(b) Let $\mathcal{A}$ be the family of all subsets A of the natural numbers N where A has the following properties: A is finite and the greatest common divisor of the elements of A is 1.

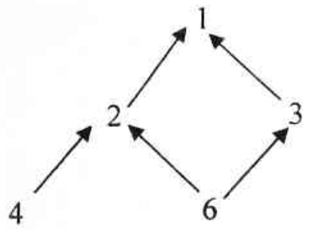
(i) State whether or not each of the following subsets of N belongs to $\mathcal{A}$:

(a) $\{2,3,8\}$ (b) $\{2,3,5,8\}$ (c) $\{2,5\}$ (d) $\{2,3,4,5, \ldots\}$

(e) $\{4,6,8\}$ (f) $\{2,3\}$.

(ii) Order $\mathcal{A}$ by set inclusion, i. e., $X \preccurlyeq Y$ if $X \subset Y$, and let $\mathcal{B}$ be the subfamily of $\mathcal{A}$ which consists of the sets in (i) which belong to $\mathcal{A}$. Construct a diagram of $\mathcal{B}$.

5.(a) Let $B = \{1, 2, 3, 4, 6\}$ be ordered as follows:



(i) Find all the minimal elements of B. (ii) Find all the maximal elements of B.

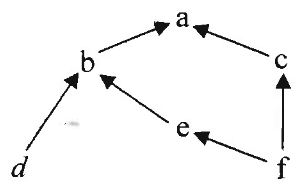
(iii) Does B have a first element? (iv) Does B have last element?

P.T.O

Let $\mathcal{B}$ be the family of all non-empty totally ordered subsets of B and let $\mathcal{B}$ be partially ordered by set inclusion.

(v) Find all the minimal elements of $\mathcal{B}$. (vi) Find all the maximal elements of $\mathcal{B}$.
(vii) Does $\mathcal{B}$ have a first element? (viii) Does $\mathcal{B}$ have last element?

(b) Define the first and last elements. Let $V = \{a, b, c, d, e, f\}$ be ordered as follows:



(i) Find all the minimal elements of V. (ii) Find all the maximal elements of V.
(iii) Does V have a first element? (iv) Does V have a last element?

Let $\mathcal{V}$ be the family of all non-empty totally ordered subsets of V and let $\mathcal{V}$ be ordered by set inclusion.

(v) Find all the minimal elements of $\mathcal{V}$. (vi) Find all the maximal elements of $\mathcal{V}$.
(vii) Does $\mathcal{V}$ have a first element? (viii) Does $\mathcal{V}$ have a last element?

6.(a)Let A be an ordered set and, for any element $a \in A$, let $S(a)$ be the set of elements which precede a, i.e., $S(a) = \{x/x \in A, x \lesssim a\}$. Let $\mathcal{A} = \{S(a)\}_{a\in A}$, the family of all sets $S(a)$, be partially ordered by set inclusion. Prove that A is similar to $\mathcal{A}$.

(b) Give an example of an ordered set $X = (A, R)$ which is similar to $Y = (A, R^{-1})$, the set A with the inverse order.

****