**Bago University**

**Department of Mathematics**

**First Semester Examination, March 2019**

**Third Year (B.Sc)**

**(Mathematics Specialization)**

**Math 3104**

**Differential Geometry**

**Time Allowed: (3) hours**

**Answer All Questions.**

1.( a ) Show that the representation $\bar{x} = x_1\hat{e}_1 + x_2\hat{e}_2$, where $x_1 = \cos\theta(2\cos\theta - 1), x_2 = \sin\theta(2\cos\theta - 1), 0 \le \theta \le 2\pi$, is regular.

( b ) Compute the length of the arc $\bar{x} = e^t \cos t\ \hat{e_1} + e^t \sin t\ \hat{e_2} + e^t\hat{e_3},\quad 0 \le t \le \pi.$

2. Find the arc length as a function $\theta$ along the epicycloids,

$$x_1 = (r_0 + r)cos\theta - rcos\left(\frac{r_0+r}{r}\theta\right), x_2 = (r_0 + r)sin\theta - rsin\left(\frac{r_0+r}{r}\theta\right).$$

3. ( a ) Find the intersection of the $x_1x_2$ plane and the tangent lines to the helix

$$\bar{x} = (cost)\hat{e_1} + (sint)\hat{e_2} + t\hat{e_3}, (t > 0).$$

( b ) Find the equations of the tangent line and the equation of normal plane to the curve $\bar{x} = (1+t)\hat{e}_1 - t^2\hat{e}_2 + (1+t^3)\hat{e}_3$ at $t = 1$.

4. ( a ) Find the torsion along the curve $\bar{x} = a(\cos t)\hat{e}_1 + a(\sin t)\hat{e}_2 + bt\hat{e}_3$, $a > 0, b \ne 0$.

( b ) Show that the curve $\bar{x} = t\hat{e}_1 + t^2\hat{e}_2 + t^3\hat{e}_3$ has 6-points contact with the paraboloid $x_1^2 + x_3^2 - x_2 = 0$.

5. ( a ) Find the curvature vector, radius of curvature along the circle of radius $a$ $\bar{x} = a(\cos t)\hat{e}_1 + a(\sin t)\hat{e}_2, (a > 0)$.

( b ) Show that the equations $k = \frac{1}{s}, \tau = 0, s > 0$ are the intrinsic equations of a logarithmic spiral.

6. ( a ) Find the equation of the involute of the circle $\bar{x} = a(\cos\theta)\hat{e}_1 + a(\sin\theta)\hat{e}_2, a > 0$.

( b ) Show that $\bar{x} = u\hat{e}_1 + v\hat{e}_2 + \sqrt{1-(u^2+v^2)}\ \hat{e}_3$, $u^2 + v^2 < 1$ is a regular parametric representation of class $C^\infty$.

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