

**Bago University**  
**Department of Mathematics**  
**First Semester Examination, March 2019**

**Third Year (B.Sc)**  
**(Mathematics Specialization)**

**Math 3105**  
**Tensor Analysis**  
**Time Allowed: (3) hours**

**Answer All Questions.**

1. (a) Prove that a cylindrical coordinate system is orthogonal.

(b) Prove  $\frac{d}{dt} \hat{e}_\rho = \dot{\phi} \hat{e}_\phi, \frac{d}{dt} \hat{e}_\phi = -\dot{\phi} \hat{e}_\rho$  where dots denote differentiation with respect to time  $t$ .

2. (a) Find the volume element  $dV$  in (i) cylindrical, (ii) spherical and (iii) parabolic cylindrical coordinates.

(b) Show that in orthogonal coordinates (i)  $\vec{\nabla} \cdot (A_1 \hat{e}_1) = \frac{1}{h_1 h_2 h_3} \frac{\partial}{\partial u_1} (A_1 h_2 h_3)$  and

$$(ii) \vec{\nabla} \times (A_1 \hat{e}_1) = \frac{\hat{e}_2}{h_3 h_1} \frac{\partial}{\partial u_3} (A_1 h_1) - \frac{\hat{e}_3}{h_1 h_2} \frac{\partial}{\partial u_2} (A_1 h_1).$$

3. (a) Express  $\text{div } \underline{A} = \underline{\nabla} \cdot \underline{A}$  in orthogonal coordinates.

(b) Write Laplace's equation in parabolic cylindrical coordinates.

4. (a) Write the terms in each of the following indicated sums.

(i)  $a_{jk} x^k$ , (ii)  $A_{pq} A^{qr}$ .

(b) Write the law of transformation for the tensors

(i)  $A_k^j$ , (ii)  $B_m^{jk}$ , (iii)  $C_{mn}$ , (iv)  $A_m$ .

5. (a) Let  $A_{rst}^{pq}$  be a tensor. (i) Choose  $p = t$  and show that  $A_{rsp}^{pq}$ , where the summation convention is employed, is a tensor. What is its rank? (ii) Choose  $p = t$  and  $q = s$  and show similarly that  $A_{rqp}^{pq}$  is a tensor. What is its rank?

(b) Determine the conjugate metric tensor in spherical coordinates.

6. (a) Express the relationship between the associated tensors,

(i)  $A^{jkl}$  and  $A_{pqr}$ , (ii)  $A_{j,l}^k$  and  $A^{qkr}$ , (iii)  $A_{.q.t}^{p.rs}$  and  $A_{jqk}^{.sl}$ .

(b) (i) Show that  $L^2 = g_{pq} A^p A^q$  is an invariant. (ii) Show that  $L^2 = g^{pq} A_p A_q$ .

\*\*\*\*\*