Bago University

Department of Mathematics

Second Semester Examination, September 2019

Third Year(B.Sc) **Math-3108**

(Mathematics Specialization) **Linear Algebra III**

Answer All Questions. **Time Allowed: (3) hours**

1.(a) Let V be the space of continuous real-valued function on the interval $[0,1]$. If $f, g \in V$, we define $\langle f,g\rangle = \int_0^1 f(t)g(t)dt$. Show that this is a positive definite scalar product.

(b) Let $v_1, \ldots, v_n$ be vectors which are mutually perpendicular, and such that $\|v_i\| \neq 0$ for all i. Let v be an element of V, and let c_i be the component of v along v_i. Let $a_1, \ldots, a_n$ be numbers. Prove that

$$\left\| v - \sum_{k=1}^{n} c_k v_k \right\| \le \left\| v - \sum_{k=1}^{n} a_k v_k \right\|.$$

2.(a) Find an orthonormal basis for the subspace of R^4 generated by the vectors $(1,2,1,0)$ and $(1,2,3,1)$.

(b) Let V be a vector space over C with a positive definite Hermitian product. Prove that Schwarz inequality $|\langle v,w\rangle| \le \|v\|\,\|w\|$ where $v, w \in V$.

3.(a) Let $V = C^3$. If $X = (x_1, x_2, x_3)$ and $Y = (y_1, y_2, y_3)$ are vectors in C^3, their product defined by $\langle X,Y\rangle = x_1\bar{y}_1 + x_2\bar{y}_2 + x_3\bar{y}_3$. Show that this product is a positive definite Hermitian product.

(b) Find an orthogonal basis for the space C^2 over C, if the scalar product is given by $X \cdot Y = x_1y_1 - i\,x_2y_1 - i\,x_1y_2 - 2x_2y_2$.

P.T.O.

4.(a) (i) Let V be the vector space of finite dimension n over the field K. Let ϕ be a functional on V and assume $\phi \neq 0$. What is the dimension of the kernel of ϕ?

(ii) Find the dimension of the space of solutions of the following systems of equations. Also find a basis for this space of solutions.

$$4x + 7y - \pi z = 0$$
$$2x - y + z = 0.$$

(b) Let x_1, x_2, x_3 be the coordinates of a vector X, and y_1, y_2, y_3 be the coordinates of a vector Y. Express in terms of these coordinates the bilinear form associated with the quadratic form $f(X) = x_1^2 - 3x_1x_2 + 4x_2^2$.

5.(a) Let a matrix A be skew-symmetric. Show that $\text{Det}(A)$ is zero if A is an $n \times n$ matrix and n is odd.

(b) Show that the absolute value of the determinant of a real unitary matrix is equal to 1. Conclude that if A is real unitary, then $\text{Det}(A) = 1$ (or) -1.

6.(a) (i) Let A be an invertible symmetric matrix. Show that A^{-1} is symmetric.

(ii) Let A be a hermitian matrix. Show that ${}^tA$ and $\bar{A}$ are hermitian. If A is invertible, show that A^{-1} is hermitian.

(b) Let A be a real unitary matrix.

(i) Show that ${}^tA$ is unitary.

(ii) Show that A^{-1} exists and is unitary.

(iii) If B is real unitary, show that AB is unitary, and that $B^{-1}AB$ is unitary.

****