Bago University

Department of Mathematics

Second Semester Examination, September 2019

Third Year (B.Sc) **Math-3109**

(Mathematics Specialization) **Mechanics**

Answer All Questions. **Time Allowed: (3) Hours**

1.(a)A bullet of mass m, moving with velocity v, strikes a block of mass M, which is free to move in the direction of the motion of the bullet, and is embedded in it. Show that the loss of kinetic energy is $\frac{M m v^2}{2(M+m)}$. If the block is afterwards struck by an equal bullet moving in the same direction with the same velocity, show that there is further loss of energy equal to $\frac{M^2 m v^2}{2(M+2m)(M+m)}$.

(b)Two weights of 9 lb and 7 lb, are fastened to the ends of a light thread which passes over a smooth pulley, the two portions of the string being vertical. The system is released from rest, and after moving for 2 s, a weight of 5 lb, at rest is suddenly attached to the 7 lb weight. Find when the system will come to test again. How far will the original weights have moved altogether?

2.(a) A spheres of masses1 lb, moving at 10 $ft\,s^{-1}$, overtake another sphere of mass 5 lb moving in the same line at 3 $ft\,s^{-1}$. Find the loss of kinetic energy during impact, and show that the direction of motion of the first sphere is reversed. (coefficient of restitution = 0.75).

(b)Let m_1, m_2 be the masses, u_1 and u_2 be their velocities before impact, v_1 and v_2 be their velocities after impact, and e be the coefficient of restitution. Show that the loss of kinetic energy due to direct impact is $\frac{1}{2}\frac{m_1 m_2}{m_1+m_2}(u_1-u_2)^2(1-e^2)$.

P.T.O.

3.(a) Two equal balls are lying in contact on a smooth table, and a third equal ball, moving along their common tangent, strikes them simultaneously. Provethat $\frac{3}{5}(1-e^2)$ of its kinetic energy is lost by the impact, e being the coefficient of restitution for each pair of balls.

(b) A uniform lamina has the shape of a quadrant of a circle. Find the product of inertia of the lamina about the bounding edges.

4.(a) Find the moment of inertia of a rectangular lamina, of mass M and sides $2a$, $2b$ about an axis through one corner perpendicular to the plane of the lamina.

(b) A flywheel of weight 1 ton and radius of gyration 3 ft 6 in is rotating once every second. What is its kinetic energy and how long will it take to come to rest under a frictional torque round the axis of 40 lb ft?

5.(a) A uniform circular cylinder of mass M, can rotate freely about its axis, which is fixed in a horizontal position; a light inextensible string is coiled round the cylinder and carries at its free end a particle of mass m. If the system is allowed to move, show that the particle will descend with uniform acceleration $\frac{2mg}{M+2m}$.

(b) Show that the acceleration of a thin uniform circular ring, rolling down a plane of inclination α which is rough enough to prevent sliding, is $\frac{1}{2}g\sin\alpha$. Show also that the least coefficient of friction necessary to prevent sliding is $\frac{1}{2}\tan\alpha$.

6.(a) A particle acted on by a central attractive force μu^3 is projected with a velocity $\frac{1}{a}\sqrt{\mu}$ at an angle of $\frac{1}{4}\pi$ with its initial distance a from the centre of force: show that its orbit is the equiangular spiral $r = ae^{-\theta}$.

(b) The law of force is μu^5 and a particle is projected from an apse at distance a. Find the orbit when the velocity of projection is $\frac{\sqrt{\mu}}{a^2\sqrt{2}}$.

****