Bago University
Department of Mathematics
Second Semester Examination, September 2018

Third Year(B. Sc) | **Math 3111**
(Mathematics Specialization) | **Complex Variables II**
Time Allowed: (3) Hours

Answer ALL questions.

1. (a) Determine the angle of rotation at the point $z_0 = 2 + i$ when $w = z^2$, and illustrate it for some particular curve. Show that the scale factor at that point is $2\sqrt{5}$.

   (b) Find the points at which the function $f(z) = x^2 - iy^2$ is

   (i) differentiable, (ii) analytic, (iii) conformal.

2. (a) If $u(x,y) = xy$, find its harmonic conjugate $v(x,y)$ and $f(z)$ in terms of z.

   (b) Find the harmonic conjugate of $u(x,y) = x^3 - 3xy^2$, by taking $(x_0, y_0) = (0,0)$ along the straight line joining $(0,0)$ to (x,y) and then find the value of $f(z)$ in terms of z.

3. (a) Let $f(z) = z^2$ and let $h(u,v) = e^{-v}\sin u$. Show that $h(u,v)$ is harmonic and find $H(x,y)$.

   (b) The transformation $w = iz^2$ maps the half line $y = x,\ x > 0$ onto the negative u- axis, $u \le 0, v = 0$ and the function $h(u,v) = v + 2$ and H assumes the value $h = 2$ on the negative u- axis. Write an explicit expression for the function $H(x,y)$. Then illustrate the theorem by showing directly that $H = 2$ along the half line $y = x,\ x \ge 0$.

4. Find the bounded steady temperatures $T(x,y)$ in the semi-infinite solid $y \ge 0$ if $T = 0$ on the part $x < -1,\ y = 0$ of the boundary, and $T = 1$ on the part $x > 1,\ y = 0$ and if the strip $-1 < x < 1, y = 0$ of the boundary is insulated.

5. Let $f(w) = Aw$ and the transformation $w = f(z) = z + \frac{1}{z}$. A long circular cylinder is of unit radius is placed in large body of fluid flowing with a uniform velocity. Prove that the top view will be the circle $x^2 + y^2 = 1$.

6. Find a function harmonic inside the unit circle $|z| = 1$ and taking the prescribed values given

   by $F(\theta) = \begin{cases} 1, & 0 < \theta < \pi \\ 0, & \pi < \theta < 2\pi \end{cases}$ on its circumference.

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