Bago University

Department of Mathematics

First Semester Examination, March 2019

Fourth Year (B.Sc)

(Mathematics Specialization)

Math 4103

Linear Programming

Time Allowed: (3) hours

Answer All Questions.

1.(a) Convert the following problems to standard form:

(i) minimize $x+2y+3z$

subject to $2 \le x+y \le 3$

$4 \le x+z \le 5$, $x \ge 0,\ y \ge 0,\ z \ge 0.$

(ii) minimize $x+y+z$

subject to $x+2y+3z=10,\ x \ge 1,\ y \ge 2,\ z \ge 1.$

(b) A manufacturer wishes to produce an alloy that is, by weight, 30% metal *A* and 70% metal *B*. Five alloys are available at various prices as indicated below:

Alloy	1	2	3	4	5
% A	10	25	50	75	95
% B	90	75	50	25	5
Price/lb	\$5	\$4	\$3	\$2	\$1.50

The desired alloy will be produced by combining some of the other alloys. The manufacturer wishes to find the amounts of the various alloys needed and to determine the least expensive combination. Formulate this problem as a linear program.

2.(a) How many basic feasible solutions are there to the following problem?

$$x_1+2x_2-x_3+x_4=3$$
$$2x_1+4x_2+x_3+2x_4=12$$
$$x_1+4x_2+2x_3+x_4=9,\ x_i \ge 0,\ i=1,2,3,4.$$

Find a basic feasible solution to the problem which satisfies $x_1 = 0$.

(b) Solve the following linear programming problem by simplex method.

Maximize $3x_1 + x_2 + 3x_3$

subject to $2x_1 + x_2 + x_3 \le 2$

$x_1 + 2x_2 + 3x_3 \le 5$

$2x_1 + 2x_2 + x_3 \le 6$, $x_1 \ge 0,\ x_2 \ge 0,\ x_3 \ge 0.$

3.(a) Find the optimal solution of the following LP problem by using the two-phase simplex procedure.

Minimize $4x_1 + x_2 + x_3$

subject to $2x_1 + x_2 + 2x_3 = 4$

$3x_1 + 3x_2 + x_3 = 3,\ x_i \ge 0,\ i=1,2,3.$

P.T.O.

(b) Find the dual problem of the following linear program:

minimize $18x_1 + 12x_2 + 2x_3 + 6x_4$

subject to $3x_1 + x_2 - 2x_3 + x_4 = 2$

$x_1 + 3x_2 \quad - x_4 = 2,\ x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0,\ x_4 \geq 0.$

4. Using the revised simplex method find a basic feasible solution to

$$x_1 + 2x_2 - x_3 + x_4 = 3$$
$$2x_1 + 4x_2 + x_3 + 2x_4 = 12$$
$$x_1 + 4x_2 + 2x_3 + x_4 = 9\ ,\ x_i \geq 0,\ i = 1,\ 2,\ 3,\ 4.$$

5.(a) Using the LP problem:

minimize $5x_1 + 12x_2 + 4x_3$

subject to $x_1 + 2x_2 + x_3 = 10$

$2x_1 - x_2 + 3x_3 = 8,\ x_i \geq 0,\ i = 1,2,3.$

(i) Find the dual problem of the above problem.

(ii) Suppose that x_2 and x_3 are positive in the optimal solution to the problem. Use the result in (i) and the complementary slackness conditions to find the optimal dual solution.

(iii) Using the result (ii), determine the optimal solution to the problem.

(b) Find the dual problem of the following linear program and find its optimal solution.

Minimize $2x_1 + x_2$

subject to $2x_1 - x_2 - x_3 \geq 3$

$x_1 - x_2 + x_3 \geq 2\ ,\ x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0.$

6.(a) Solve the following problem by using the dual simplex method.

Minimize $3x_1 + 4x_2 + 5x_3$

subject to $x_1 + 2x_2 + 3x_3 \geq 5$

$2x_1 + 2x_2 + x_3 \geq 6,\ x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0.$

(b) Consider the problem

minimize $2x_1 + x_2 + 4x_3$

subject to $x_1 + x_2 + 2x_3 = 3$

$2x_1 + x_2 + 3x_3 = 5\ ,\ x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0.$

(i) What is the dual problem?

(ii) Note that $\lambda = (0,0)$ is feasible for the dual. Starting with this λ, solve the primal using the primal-dual algorithm.

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