Bago University

Department of Mathematics

Second Semester Examination, September 2019

Fourth Year (B.Sc) **Math- 4110**

Mathematics Specialization **Hydromechanics**

Answer All Questions **Time Allowed: (3) Hours**

1(a) At the point in an incompressible fluid having spherical polar coordinates (r, θ, ψ) , the velocity components are $[2Mr^{-3}cos\theta, Mr^{-3}sin\theta, 0]$, where M is a constant.Show that the velocity is of the potential kind.Find the velocity potential and the equations of the stream lines.

(b) AB is a tube of small uniform bore forming a quadrantal arc of a circle of radius a and centre O, OA being horizontal and OB vertical with B below O.The tube is full of liquid of density ρ, the end B being closed.If B is suddenly opened, show that initiall $\frac{du}{t} = \frac{2g}{\pi}$, where u = u(t) is the velocity, and that the pressure at a point whose angular distance from A is θ immediately drops to $\rho ga\left(sin\theta - \frac{2\theta}{\pi}\right)$ above atmospheric pressure. Prove further that when the liquid remaining in the tube substends an angle β at the centre

$$\frac{d^2\beta}{dt^2} = -\frac{2g}{a\beta}\sin^2\left(\frac{\beta}{2}\right).$$

2(a) Solid sphere is moving with constant velocity U $\hat{\imath}$ in liquid which is otherwise at rest. Find the velocity potential for this motion and find also the total K.E of the sphere and fluid.

(b) A simple source, strength m, is fixed at the origin O in a uniform steam of incompressible fluid moving with velocity U $\hat{\imath}$. Show that the velocity potential φ at any point P of the stream is $\left(\frac{m}{r}\right) - Urcos\theta$, where OP = r and θ is the angle $\overline{OP}$ makes with the direction $\hat{\imath}$. Find the differential equation of the streamlines and show that they lie on the surfaces $Ur^2\sin^2\theta - 2m\cos\theta = \text{const.}$

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3(a) Show that the specific gravity of a mixture of n liquids is greater when equal volumes are taken than when equal weights are taken, assuming no change in volume as the result of mixting.

(b) Three liquids, whose densities are in A.P, fill a semi-circular tube whose bounding diameter is horizontal. Prove that the depth of one of the common surfaces is double that of the other.

4(a) The lower ends of two vertical tubes, whose cross sections are 1 and 0.1 sq inch respectively, are connected by a tube. The tube contains mercury of sp.gr 13.596. How much water must be poured into the larger tube to raise the level in the smaller tube by one inches?

(b) A square plate, whose edge is 8 inches,is immersed in water,its upper edge being horizontal at a depth of 12 inches below the surface of the water. Find the thrust of the water on the surface of the plate when it is inclined at 45° to the horizon; the mass of a cu.ft of water being 64lbs.

5(a) A square plate, whose edge is 8 inches,is immersed in water,its upper edge being horizontal at a depth of 12 inches below the surface of the water.Find the thrust of the water on the surface of the plate when it is inclined at 45° to the horizon; the mass of a cu.ft of water being 64lbs.

(b) ABC is a triangular lamina vertically immersed in a fluid with A in the surface and BC horizontal. M and N are the mid points of AB and AC. Prove that the depth of the centre of pressure of the area MNCB is $\frac{45}{56}d$, where d is the depth of B.

6(a) A vessel in the shape of a hollow hemisphere surmounted by a cone is held with the axis vertical and vertex uppermost. If it be filled with a liquid so as to submerge half the axis of the cone in the liquid, and height of the cone be double the radius of its base, show that the resultant downward thrust of the liquid on the vessel is $\frac{15}{8}$ times the weight of the liquid that the hemisphere can hold.

(b) A hemispherical surface of radius a is immersed in liquid of density ρ with its centre of depth h and its base inclined at an angle θ to the horizontal. Find the resultant thrust on the curved surface and its direction.

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