Bago University

Department of Mathematics

Second Semester Examination, September 2019

Fourth Year (B.Sc) **Math 4111**

(Mathematics Specialization) **Stochastic Process II**

Time Allowed: (3) hours

Answer All Questions.

1.(a) Let X_1 and X_2 be independent exponential random variables, each having rate μ.

Let $X_{(1)} = \text{minimum}(X_1, X_2)$ and $X_{(2)} = \text{maximum}(X_1, X_2)$.

Find (i) $E[X_{(1)}]$, (ii) $\text{Var}[X_{(1)}]$, (iii) $E[X_{(2)}]$, (iv) $\text{Var}[X_{(2)}]$.

(b) Customers arrive in a certain shop according to an approximate Poisson process at a mean rate of 20 per hour. What is the probability that the shopkeeper will have to wait for more than five minutes for his first customer to arrive. ?

2.(a) Show that

$$\text{Var}(X_n) = \begin{cases} \sigma^2 \mu^{n-1} \dfrac{1-\mu^n}{1-\mu}, & \text{if } \mu \neq 1 \\ n\sigma^2, & \text{if } \mu = 1 \end{cases}$$

where μ and σ^2 are the mean and variance of the number of offspring an individual has.

(b) A student has a class on Monday, Wednesday and Friday. If he goes to class on one day, he goes to the next class with probability $\frac{1}{2}$. If he doesn't go to class that day, he goes to the next class with probability $\frac{3}{4}$. Show that this Markov chain is time reversible.

3. (a) Consider the gambler's ruin problem with $p = 0.4$ and $N = 4$ starting with 3 units

(i) Determine the expected amount of time the gambler's has 2 units.

(ii) What is the probability that the gambler ever has a fortune of 1 ?

P.T.O.

(b) For the Markov chain with states 1, 2, 3, 4 whose transition probability matrix P is as specified below. Find f_{23} and s_{23}.

$$P = \begin{Vmatrix} 0.4 & 0.2 & 0.1 & 0.3 \\ 0.1 & 0.5 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.2 & 0.1 \\ 0 & 0 & 0 & 1 \end{Vmatrix}$$

4.(a) Let X be an exponential random variable with parameter λ. Find the variance of X.

(b) If X_1 and X_2 are independent nonnegative continuous random variables, show that

$$P\{X_1 < X_2 \mid \min(X_1, X_2) = t\} = \frac{r_1(t)}{r_1(t) + r_2(t)}.$$

5.(a) If X has an exponential distribution, then, for every constant $a \geq 0$, show that

$P\{X - a \leq x \mid X \geq a\} = P\{X \leq x\}$ for all x.

(b) Let $N(t)$ be a Poisson process, prove that $E[N(t)] = \lambda t$ and $\text{Var}[N(t)] = \lambda t$.

6.(a) Let $\{N(t), t \geq 0\}$ be a Poisson process with rate λ that is independent of the nonnegative random variable T with mean μ variance σ^2. Find the Var[$N(t)$].

(b) The arrival of new telephone calls at a telephone switching office is a Poisson process $N(t)$ with an arrival rate of $\lambda = 4$ calls per second. An experiment consists of monitoring the switching office and recording $N(t)$ over a 10 seconds interval.

(i) What is the probability that no phone call in the first second of observation ?

(ii) What is the probability that exactly four calls arriving in the first second of observation?

(iii) What is the probability that exact two calls arriving in the first two seconds ?

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