Bago University
Department of Physics
First Semester Examination, March 2019

Second Year (BSc)
(Physics Specialization)

Phys 2101
Mathematical Physics
Time Allowed: (3) Hours

Answer any <u>Six</u> questions.

1 (a) Determine a unit vector perpendicular to the plane of $\vec{A}=2\hat{i}-6\hat{j}-3\hat{k}$ and $\vec{B}=4\hat{i}+3\hat{j}-1\hat{k}$.

(b) Find the angle between $\vec{A}=2\hat{i}+2\hat{j}-\hat{k}$ and $\vec{B}=6\hat{i}-3\hat{j}+2\hat{k}$.

2 (a) Can a vector have a component greater than its magnitude? Answer with some calculation. Show graphically that the addition of vectors is associative, that is, $\vec{A}+(\vec{B}+\vec{C}) = (\vec{A}+\vec{B})+\vec{C}$.

(b) How do you understand scalar or dot product? The following forces act on a particle: $\vec{F}_1 = 2\hat{i}+3\hat{j}-5\hat{k}$, $\vec{F}_2 = -5\hat{i}+\hat{j}+3\hat{k}$, $\vec{F}_3 = \hat{i}+2\hat{j}-\hat{k}$, measured in newton. Find (i) the resultant of the forces and (ii) a unit vector parallel to the resultant force.

3 (a) If $\vec{A}=A_x\hat{i}+A_y\hat{j}+A_z\hat{k}$, $\vec{B}=B_x\hat{i}+B_y\hat{j}+B_z\hat{k}$ and $\vec{C}=C_x\hat{i}+C_y\hat{j}+C_z\hat{k}$ show that

$$\vec{A}\cdot(\vec{B}\times\vec{C})=\begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}.$$

(b) Find the directional derivative of $x^2yz+4xz^2$ at (1,-2,-2) in the direction $2\hat{i}-\hat{j}-2\hat{k}$.

4 (a) If $\vec{A}=t^2\hat{i}-t\hat{j}+(2t+1)\hat{k}$ and $\vec{B}=(2t-3)\hat{i}+\hat{j}-t\hat{k}$, find (i) $\frac{d}{dt}(\vec{A}\cdot\vec{B})$, (ii) $\frac{d}{dt}(\vec{A}\times\vec{B})$.

(b) Determine the transformation from cylindrical to rectangular coordinates.

5 (a) Prove the curl of the gradient of a scalar function is zero. What is the greatest rate of increase of $u = xyz^2$ at (1,0,3)?

(b) Show that (i) $\vec{\nabla}\cdot(\vec{A}+\vec{B})=\vec{\nabla}\cdot\vec{A}+\vec{\nabla}\cdot\vec{B}$ (ii) $\vec{\nabla}\cdot(\phi\vec{A})=(\vec{\nabla}\phi)\cdot\vec{A}+\phi(\vec{\nabla}\cdot\vec{A})$.

6 (a) If $\phi=3x^2-yz$ and $\vec{A}=3xyz^2\hat{i}+2xy^3\hat{j}-x^2yz\hat{k}$, find $\vec{\nabla}\times(\phi\vec{A})$ at (1,-1,1).

(b) If $\vec{A}=x\hat{i}+y\hat{j}+z\hat{k}$, evaluate $\int_V \vec{\nabla}\cdot\vec{A}\,dV$ for spherical coordinates and V is the sphere $x^2+y^2+z^2=a^2$.

7 (a) Using divergence theorem, derive (a) Gauss's law for the electric field (b) Gauss's law for the magnetic field.

(b) Find $\int_C \vec{A}\cdot d\vec{r}$ for $\vec{A}=(2x-y)\hat{i}-yz^2\hat{j}-y^2z\hat{k}$ where S is the upper half surface of the sphere $x^2+y^2+z^2=1$ and C is its boundary.

8 (a) If $\vec{F}=2xz\hat{i}-x\hat{j}+y^2\hat{k}$, evaluate $\int_V \vec{F}\,dV$ where V is the region bounded by the surface $x=0$ to $x=2$, $y=0$ to $y=6$ and $z=x^2$ to $z=4$.

(b) If $\vec{A}=4y\hat{i}+x\hat{j}+2z\hat{k}$, evaluate $\int_S \vec{\nabla}\times\vec{A}\cdot d\vec{S}$ over the hemisphere $x^2+y^2+z^2=a^2$, $z\geq 0$.