Bago University
Department of Physics
First Semester Examination, March 2019

Third Year (BSc)
(Physics Specialization)

Phys 3109
Mathematical Physics
Time Allowed: (3) Hours

Answer any Six questions.

1 (a) Describe the explanations of Hermitian matrix and Skew Hermitian matrix with their examples.

(b) Determine the values of α,β,γ when $\begin{pmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & -\beta & \gamma \end{pmatrix}$ is orthogonal.

2 (a) Write down orthogonal matrix.

Show that $\begin{bmatrix} \cos\phi & 0 & \sin\phi \\ \sin\theta\sin\phi & \cos\theta & -\sin\theta\cos\phi \\ -\cos\theta\sin\phi & \sin\theta & \cos\theta\cos\phi \end{bmatrix}$ is an orthogonal matrix.

(b) (i) If A = $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, find A^{-1}. (ii) If a matrix A satisfies a relation $A^2 + A - I = 0$, prove that A^{-1} exist and that $A^{-1} = I + A$, I is identity matrix.

3 (a) What is a complex number? Who represented a complex number in a diagram? Show that the sum and the product of a complex number and its conjugate complex are both real.

(b) Find the continued product of the five values of the expression $(1+i)^{\frac{1}{5}}$.

4 (a) If $|Z_1+Z_2| = |Z_1-Z_2|$, prove that the difference of the amplitudes of Z_1 and Z_2 is $\frac{\pi}{2}$.

(b) (i) Write down De Moivre's theorem. Prove that the general value of θ which satisfies the equation, $(\cos\theta + i\sin\theta).(\cos 2\theta + i\sin 2\theta)...(\cos n\theta + i\sin n\theta) = 1$ is $\frac{4m\pi}{n(n+1)}$, where m is any integer. (ii) If n is positive integer, prove that $(\sqrt{3}+i)^n + (\sqrt{3}-i)^n = 2^{n+1}\cos\frac{n\pi}{6}$.

5 (a) What is modules and argument of the complex number? Describe the four types of principal values of argument with their Argand diagrams.

(b) If 'n' be a positive integer, prove that $(1+i)^n + (1-i)^n = 2^{\frac{n+2}{2}}\cos\frac{n\pi}{4}$.

6 (a) State the necessary condition for f(z) to be Analytic. Show that the function $e^x(\cos y + i\sin y)$ is an analytic function, find its derivative.

(b) Show that the real and imaginary parts of the function w = log z satisfy the Cauchy-Riemann equations where z is not zero.

7 (a) State the transformation: $w = e^z$ with its mapping of simple figures.

(b) Show that under the transformation $w = \frac{1}{z}$, the image of the hyperbola x^2-y^2= 1 is the lemniscate $R^2 = \cos 2\phi$.

8 (a) What is an analytic function? Write down the necessary conditions for f(z) in terms of polar and rectangular form to be analytic at all points in the region R.

(b) If $u - v = (x-y)(x^2+4xy+y^2)$ and $f(z) = u + iv$ is an analytic function of $z = x + iy$, find f(z) in term of z.